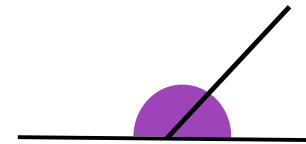
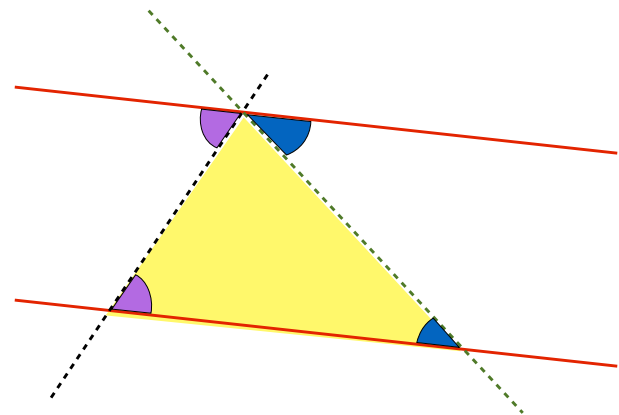
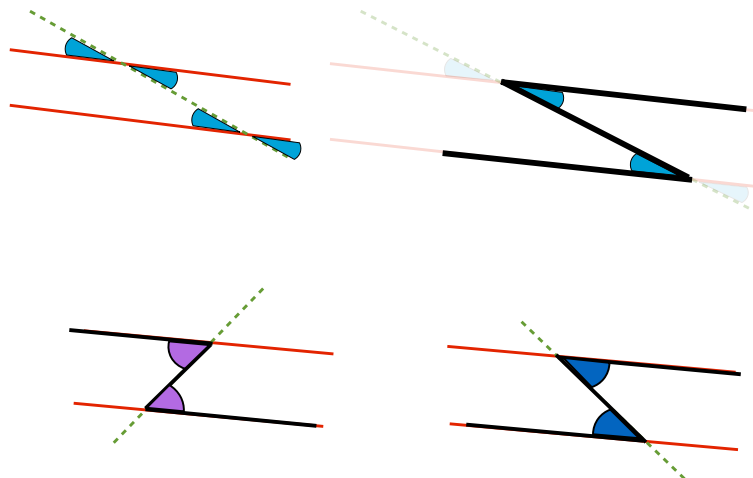


Rund um Dreieck und Kreis

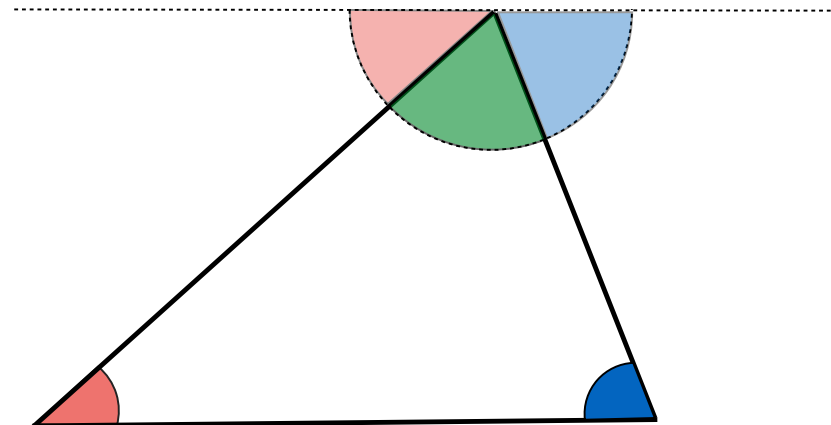


$$\alpha + \beta = 180^\circ$$

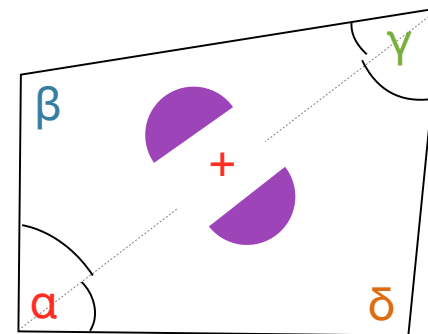


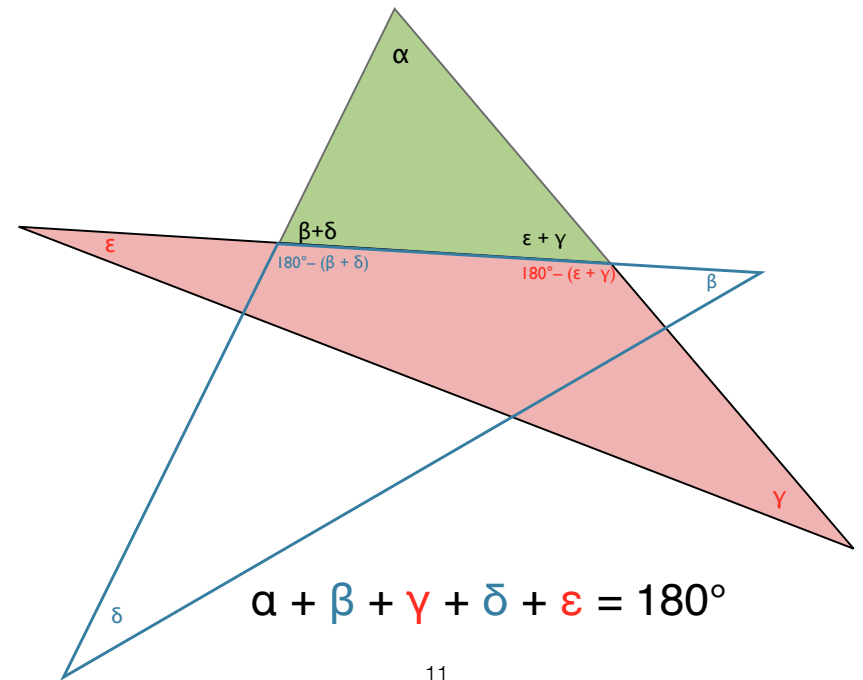


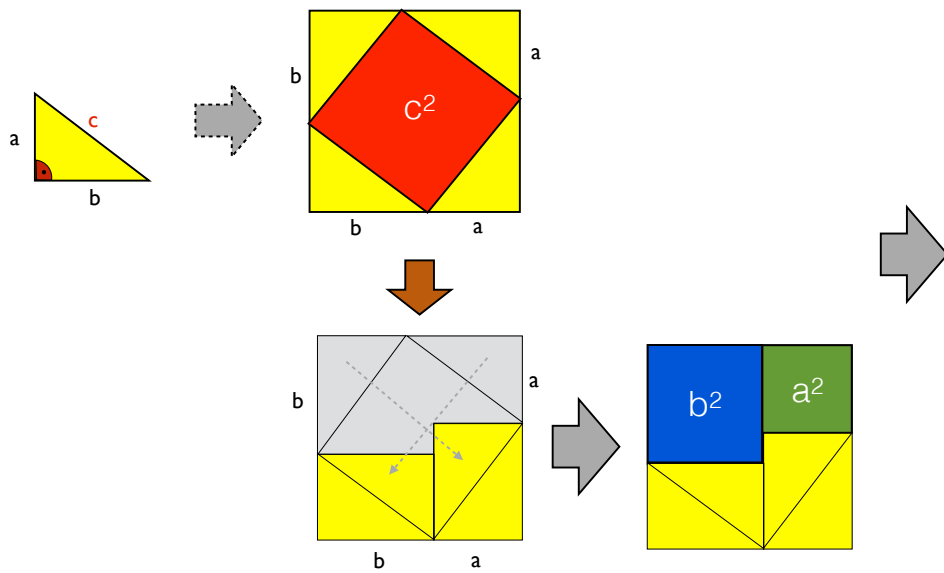
$$\alpha + \beta + \gamma = 180^\circ$$



$$\alpha + \beta + \gamma + \delta = 360^\circ$$

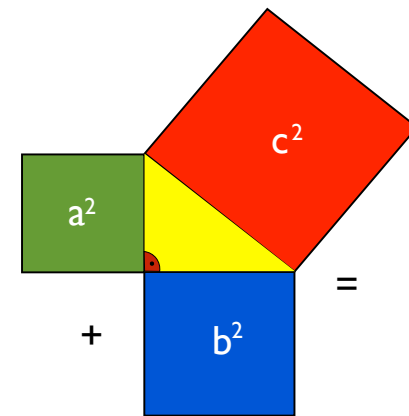






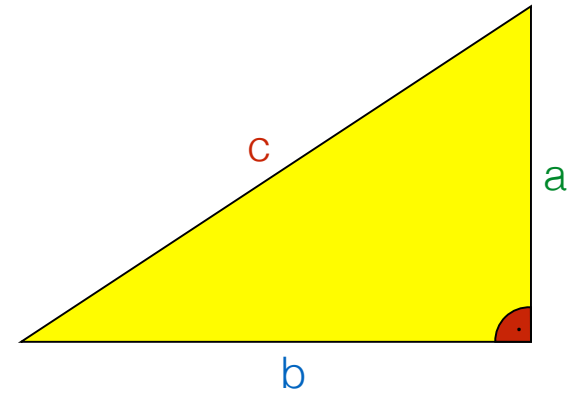
Pythagoras

12



13

$$a^2 + b^2 = c^2$$



Kannst du zuordnen?

$$\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 = \left(\frac{d}{2}\right)^2$$

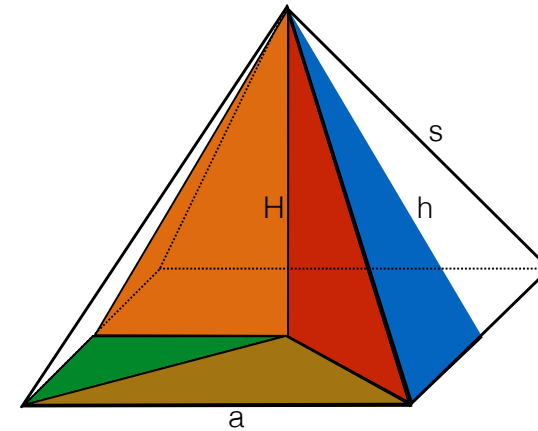
$$\left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)^2 = a^2$$

$$\left(\frac{a}{2}\right)^2 + H^2 = h^2$$

$$\left(\frac{d}{2}\right)^2 + H^2 = s^2$$

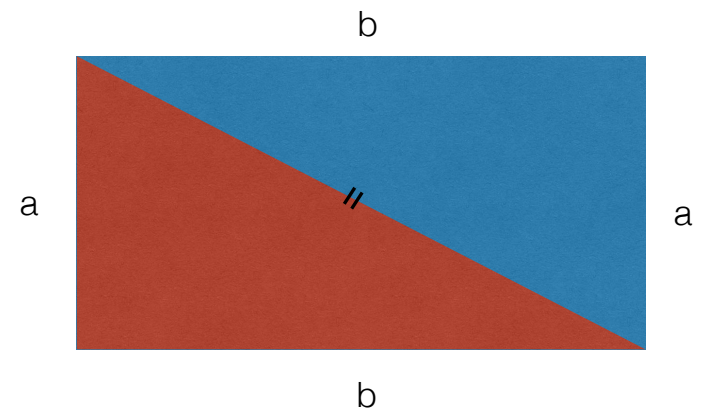
$$h^2 + \left(\frac{a}{2}\right)^2 = s^2$$

5 rechtwinklige Dreiecke

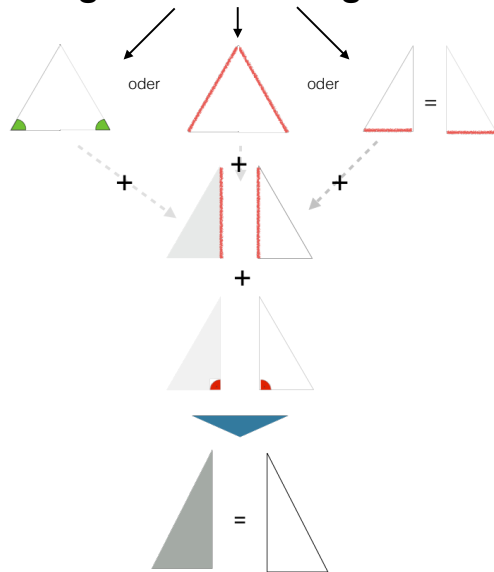


Erinnerst du dich?

Dreiecke mit 3 gleichen Eigenschaften,
sind gleich!



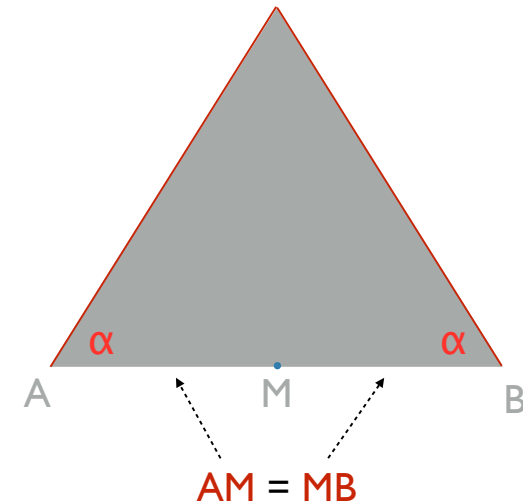
Egal wo du anfängst ...



Gleichschenkliges Dreieck

20

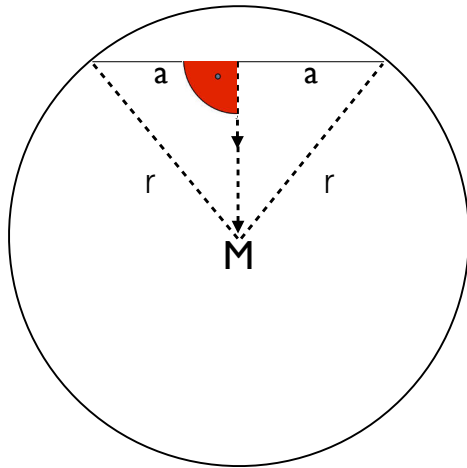
... hast du 1 Paar, dann hast du 3 Paare



21

Gleiche Seiten (r)

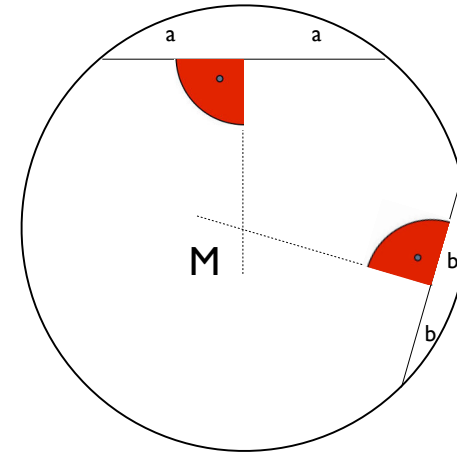
—>



Gleichschenkliges Dreieck

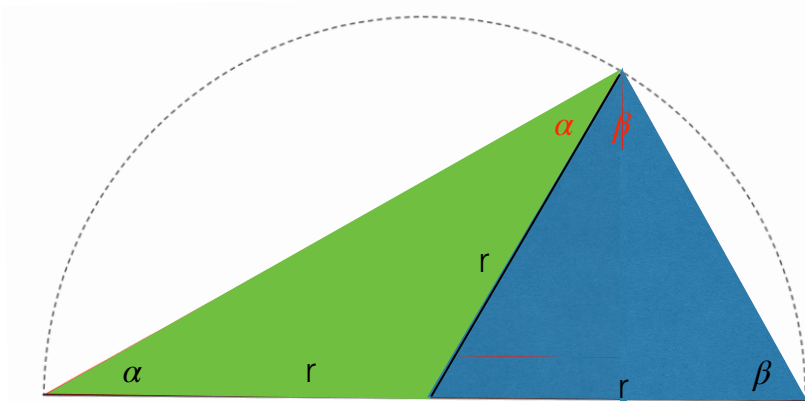
22

...die Mittelsenkrechte führt zum Mittelpunkt!



23

Gleiche Seiten (r) $\rightarrow$ gleiche Winkel (α , β)

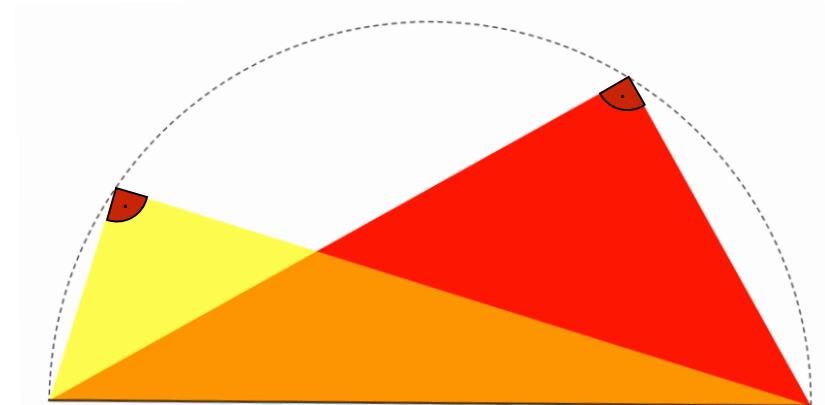


$$\alpha + \alpha + \beta + \beta = 180^\circ$$

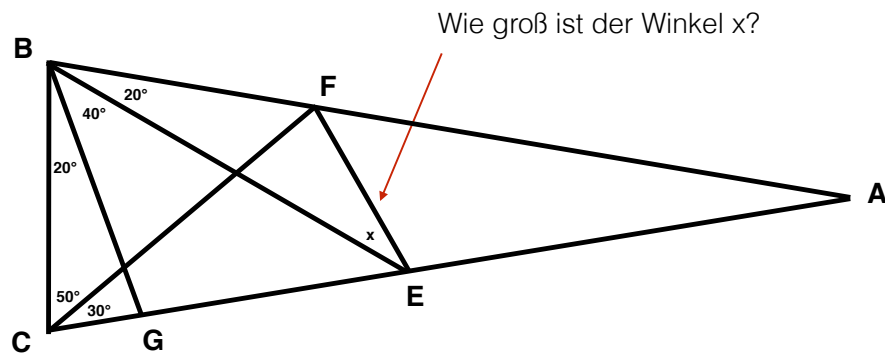
$$2(\alpha + \beta) = 180^\circ$$

$$(\alpha + \beta) = 90^\circ$$

$\rightarrow$ rechter Winkel ($\alpha + \beta$)

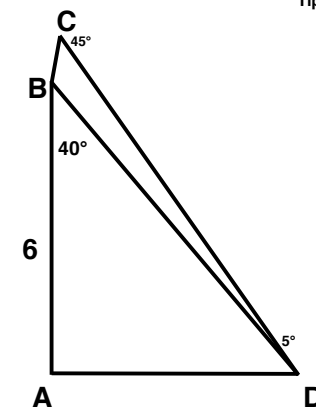


Diesen Zusammenhang hat Thales entdeckt,
deshalb heißt er **Thaleskreis**



Tip: Hangle dich von einem gleichschenkligen Dreieck zum nächsten.

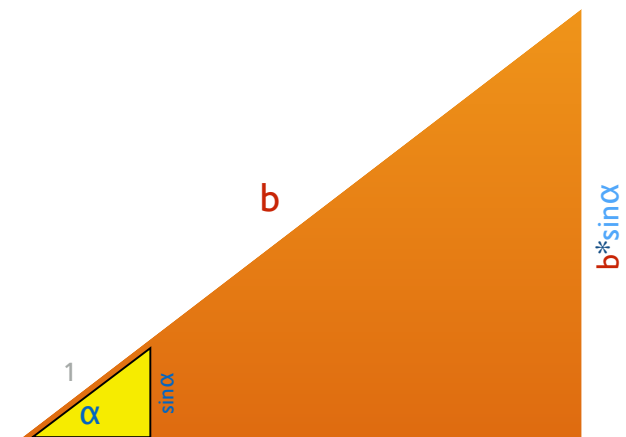
Was ist die Fläche des Vierecks ABCD?

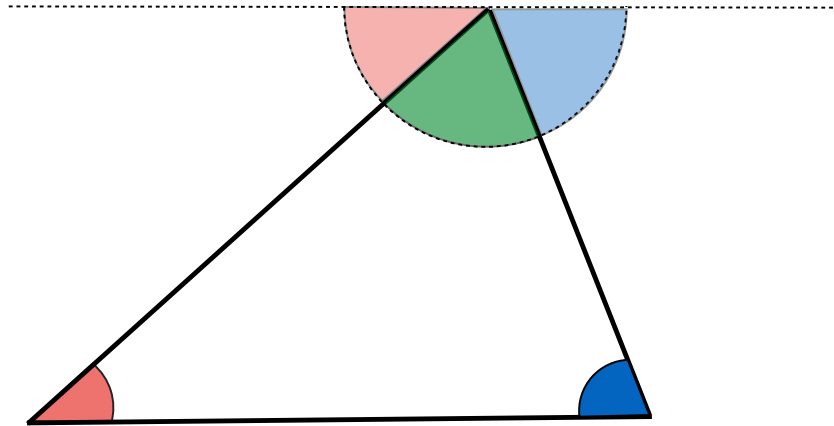


Tip: Versuch dir ein gleichschenkliges Dreieck zu basteln.

Trigonometrie

Seiten sagen mir die Winkel – und umgekehrt

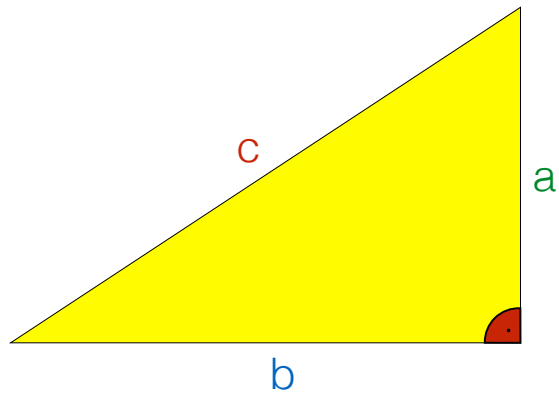




Winkelsatz

$$\alpha + \beta + \gamma = 180^\circ$$

2 Winkel → 3 Winkel



Pythagoras

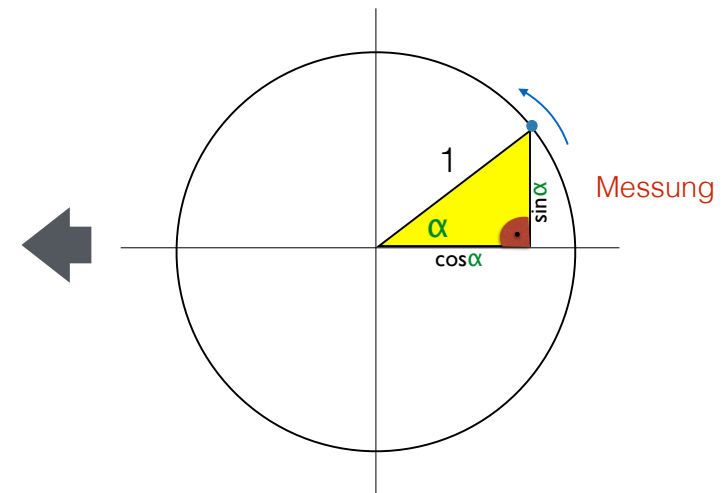
$$a^2 + b^2 = c^2$$

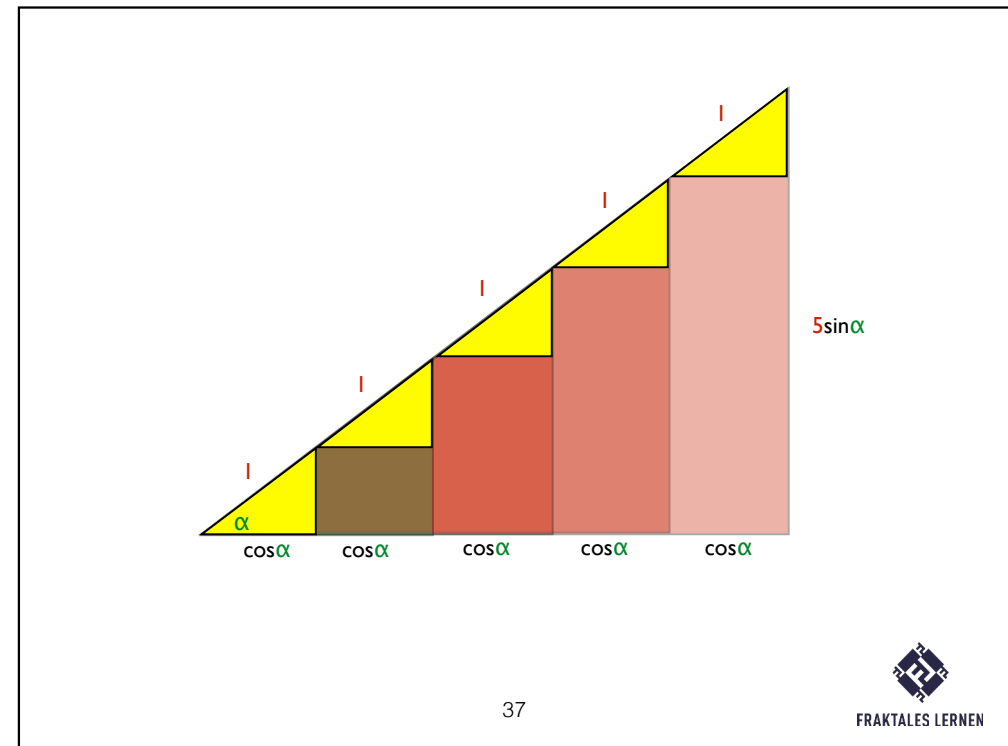
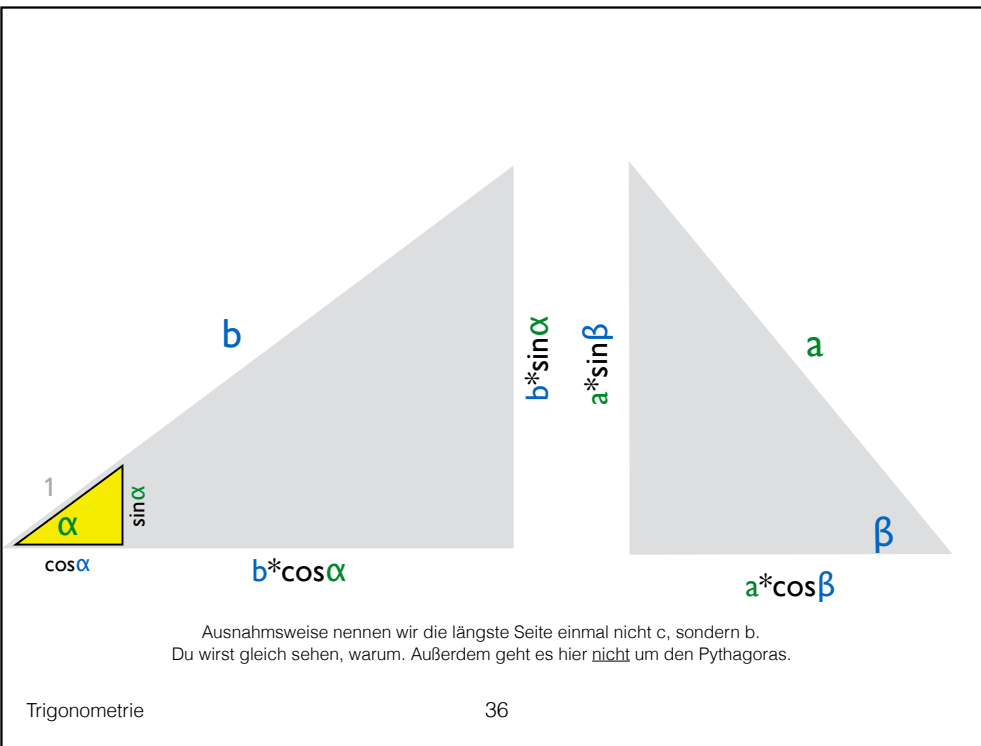
2 Seiten → 3 Seiten

Winkel Katheten

α	$\sin(\alpha)$	$\cos(\alpha)$
10°	0,174	0,985
11°	0,191	0,982
12°	0,208	0,978
13°	0,225	0,974
14°	0,242	0,970
15°	0,259	0,966
16°	0,276	0,961
17°	0,292	0,965
18°	0,309	0,951
19°	0,326	0,946
20°	0,342	0,940
21°	0,358	0,936
22°	0,375	0,927
23°	0,391	0,921
24°	0,407	0,914
25°	0,423	0,906
26°	0,438	0,899
27°	0,454	0,891
28°	0,470	0,883
29°	0,485	0,975

In einem rechtwinkligen Dreieck (mit der Hypotenuse = 1) kennt dein Taschenrechner zu jedem Winkel die Länge der Katheten. Und umgekehrt!

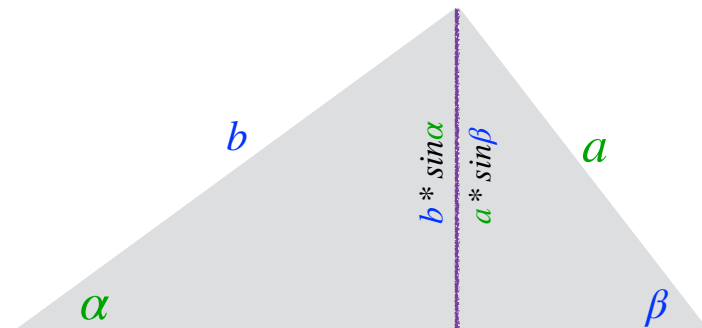


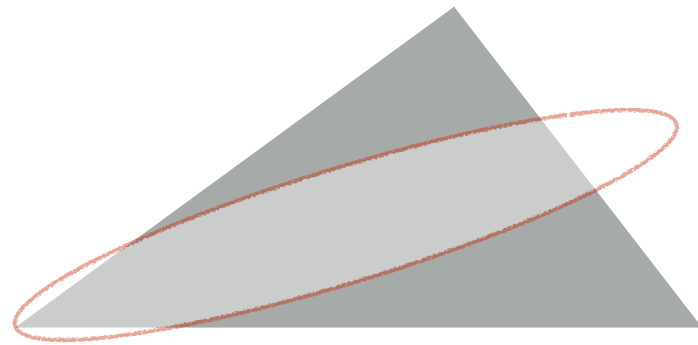


$$b * \sin\alpha = a * \sin\beta$$



$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$



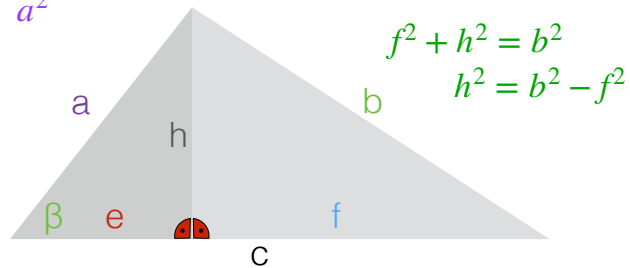


„Pärchensatz“

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b}$$

1 Pärchen → 2 Pärchen

$$e^2 + h^2 = a^2$$



$$e = a * \cos \beta$$

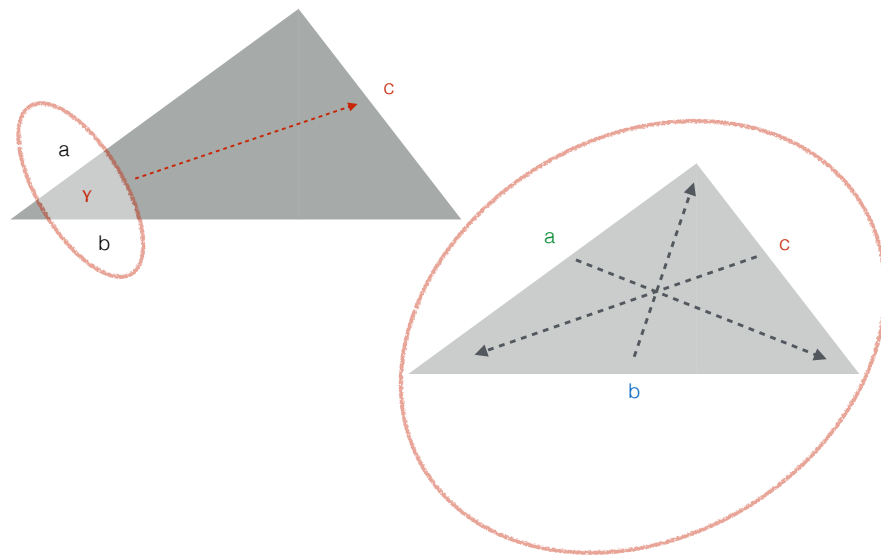
Erinnerst du dich?
Wenn nicht: sieh' dir nochmal Seite 28 an!

$$\begin{aligned} e + f &= c \\ f &= c - e \end{aligned}$$

$$\begin{aligned} f^2 + h^2 &= b^2 \\ h^2 &= b^2 - f^2 \end{aligned}$$

Alles einmal kräftig durchgemischt

$$\begin{aligned} e^2 + h^2 &= a^2 \\ e^2 + b^2 - f^2 &= a^2 \\ e^2 + b^2 - (c-e)^2 &= a^2 \\ e^2 + b^2 - (c^2 - 2ce + e^2) &= a^2 \\ \cancel{e^2} + b^2 - c^2 + 2ce - \cancel{e^2} &= a^2 \\ b^2 - c^2 + 2ce &= a^2 \\ b^2 - c^2 + 2ba * \cos \beta &= a^2 \\ b^2 &= a^2 + c^2 - 2ab * \cos \beta \end{aligned}$$



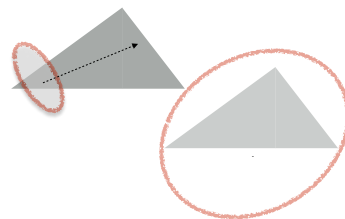
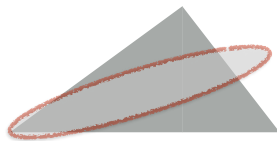
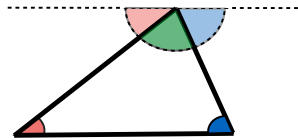
„Kein-Pärchensatz“

$$b^2 + c^2 - 2bc * \cos\alpha = a^2$$

$$a^2 + c^2 - 2ac * \cos\beta = b^2$$

$$a^2 + b^2 - 2ab * \cos\gamma = c^2$$

0 Pärchen → 1 Pärchen



$$\alpha + \beta + \gamma = 180^\circ$$

2 Winkel → 3 Winkel

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

1 Pärchen → 2 Pärchen

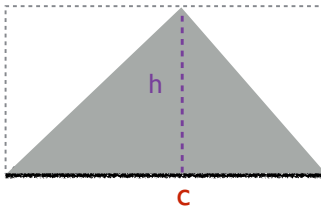
$$b^2 + c^2 - 2bc \cdot \cos \alpha = a^2$$

$$a^2 + c^2 - 2ac \cdot \cos \beta = b^2$$

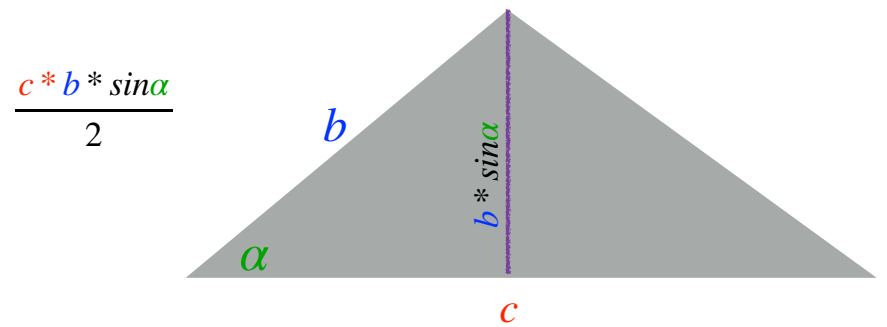
$$a^2 + b^2 - 2ab \cdot \cos \gamma = c^2$$

0 Pärchen → 1 Pärchen

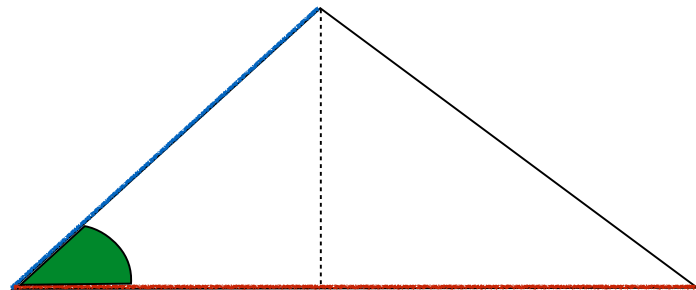




$$A = \frac{g * h}{2} =$$

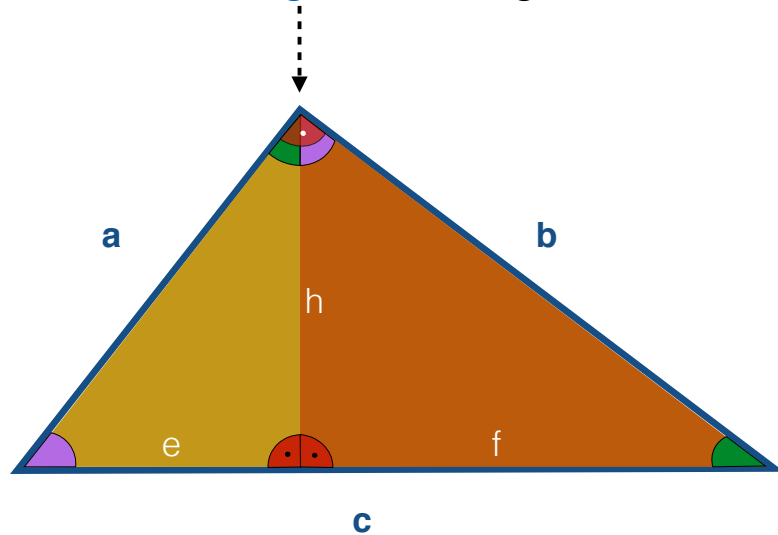


$$\frac{c * b * \sin \alpha}{2}$$



$$A = \frac{c * b * \sin \alpha}{2}$$

Jedes rechtwinklige Dreieck birgt 2 weitere ...



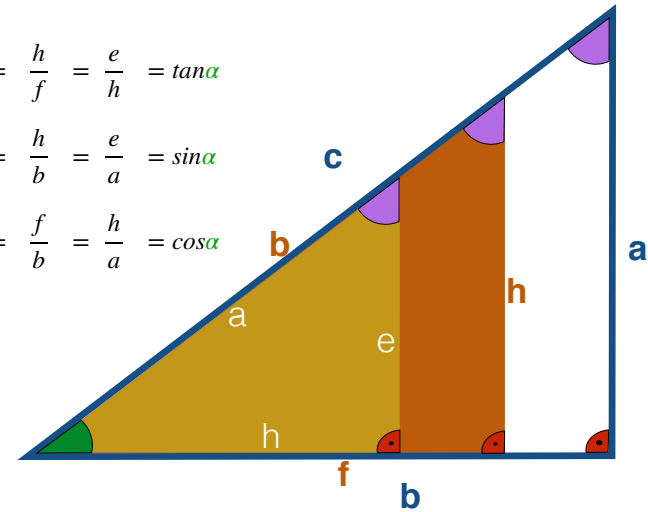
52

... dem ersten ähnliche rechtwinklige Dreiecke

$$\frac{a}{b} = \frac{h}{f} = \frac{e}{h} = \tan \alpha$$

$$\frac{a}{c} = \frac{h}{b} = \frac{e}{a} = \sin \alpha$$

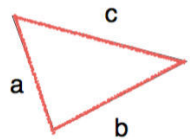
$$\frac{b}{c} = \frac{f}{b} = \frac{h}{a} = \cos \alpha$$



53

In welcher Reihenfolge und mit welchem Satz bestimmst du die fehlenden Größen?

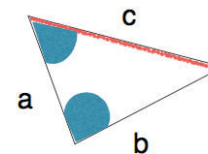
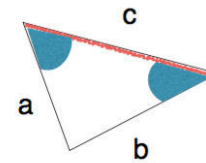
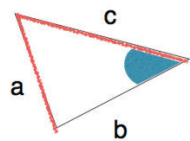
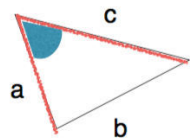
(Die roten Seiten und blauen Winkel sind bekannt)



w/s	Satz
1.	
2.	
3.	

w/s	Satz
1.	
2.	
3.	

w/s	Satz
1.	
2.	
3.	

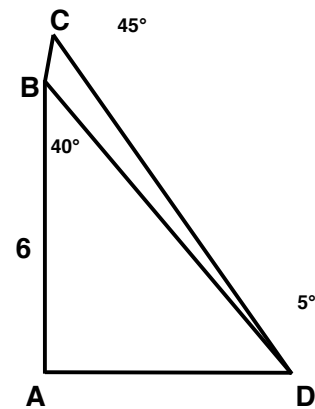


w/s	Satz
1.	
2.	
3.	

w/s	Satz
1.	
2.	
3.	

Das Rätsel kennst du.
Beim ersten mal halfen gleichschenklige Dreiecke.
Versuche es jetzt einmal mit Trigonometrie!

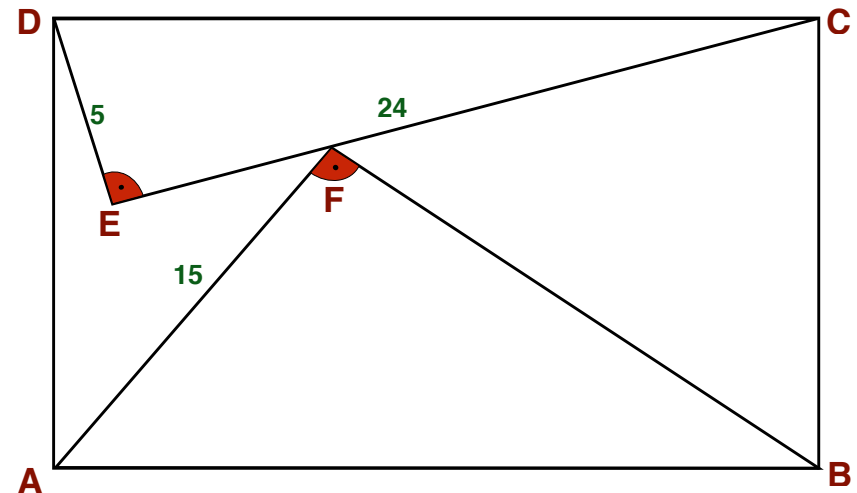
Was ist die Fläche des Vierecks ABCD?

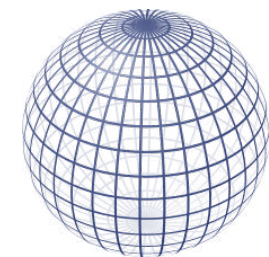
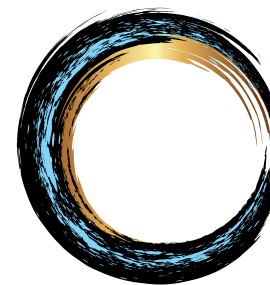


Es gibt 2 Wege:

- a) **Trigonometrie**
Tip: finde die Winkel bei B und C
- b) **Kombination aus Pythagoras und „ähnliche Dreiecke“**
Tip: finde $CD = AB$
finde BF
ziehe eine Linie parallel zu AD durch F
die letzten beiden Schritte darfst du alleine gehen

Wie lang ist die Seite AD ?





Du hast einen Namen, ...

$\pi = 3.1415926535\ 8979323846\ 2643383279\ 5028841971\ 6939937510\ 5820974944\ 5923078164\ 0628620899\ 8628034825\ 3421170679\ 8214808651\ 3282306647\ 0938446095\ 5058223172\ 5359408126\ 4811174502\ 8410270193\ 8521105559\ 6446229489\ 5493038196\ 4428810975\ 6659334461\ 2847564823\ 3786783165\ 2712019091\ 4564856692\ 3460348610\ 4543266482\ 1339360726\ 0249141273\ 7245870066\ 0631558817\ 4881520920\ 9628292540\ 9171536436\ 7892590360\ 0113305305\ 4882046652\ 1384146951\ 9415116094\ 3305727036\ 5759591953\ 0921861173\ 8193261179\ 3105118948\ 0744623799\ 6274956735\ 1885752724\ 8912279381\ 8301194912$

9833673362 4406566430 8602139494 6395224737 1907021798 6094370277 0539217176 2931767523 8467481846 7669405132
0005681271 4526356082 7785771342 7577896091 7363717872 1468440901 2249534301 4654958537 1050792279 6892589235
4201995611 2129021960 8640344181 5981362977 4771309960 5187072113 4999999837 2978049951 0597317328 1609631859
5024459455 3469083026 4252230825 3344685035 2619311881 7101000313 7838752886 5875332083 8142061717 7669147303
5982534904 2875546873 1159562863 8823537875 9375195778 1857780532 1712268066 1300192787 6611195909 2164201989

3809525720 1065489863 2788659361 5338182796 8230301952 0353018529 6899577362 2599413891 2497217752 8347913151
5574887242 4541506959 5082953111 6861727855 8890750983 8175463746 4939319255 0604009277 0167113900 9848824012
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9331367702 8989152104 7521620569 4602405803 8150193511 2533624300 3587644024 7496473263 9141992726 0426992279
6782384781 63600039417 2164121952 4586315030 2861829745 5570674983 8505494588 886926956 9092721079 7509302955

3211653449 8720275596 0236480665 4991198818 3479775356 6369807426 5425278625 5181841757 4672890977 7727938000
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4547762416 8625189835 6948556209 9219222184 2725502542 5688767179 0494601653 4668049886 2723279178 6085784383
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0674427862 2039194945 0471237137 8696095636 4371917287 4677646575 7396241389 0865832645 9958133904 7802759009

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4962524517 4939965143 1429809190 652509372 2169646151 5709858387 4105978859 5977297549 8930161753 9284681382
6868386894 2774155991 8559252459 5395943104 9972524680 8459872736 4469584865 3836736222 6260991246 0805124388
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0168427394 5226746767 8895252138 5225489546 6672782398 6456596116 3548862305 7745649803 5593634568 1743241125

1507606947 9451096596 0940252288 7971089314 5669136867 2287489405 6010150330 8617928680 9208747609 1782493858
9009714909 6759852613 6554978189 3129784821 6829898487 2265880485 7564014270 4775551323 7964145152 3746234364
5428584447 9526586782 1051141354 7357395231 1342716610 2135969536 2314429524 8493718711 0145765403 5902799344
8374200731 0578539062 1983874478 0847848968 3321445713 8687519435 0643021845 3191048481 0053706146 8067491927
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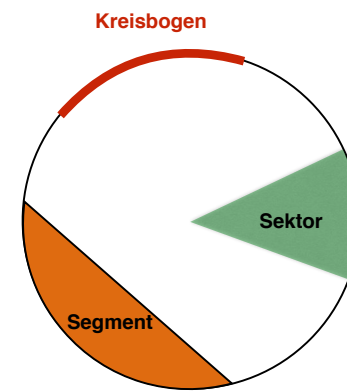
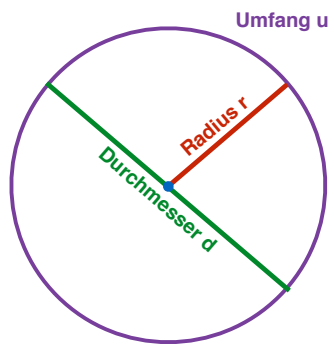
5679452080 9514655022 5231603881 9301420937 6213785595 6638937787 0830390697 9207734672 2182562599 6615014215
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1005508106 6587699981 6357473638 4052571459 1028970641 4011097120 6280439039 7595156771 5770042033 7869396007
2305587631 7635942187 3125147120 5329281918 2618612586 7321579198 4148488291 6447060957 5270695722 0917567116
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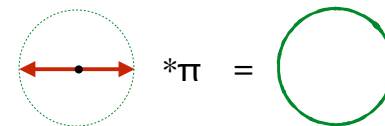
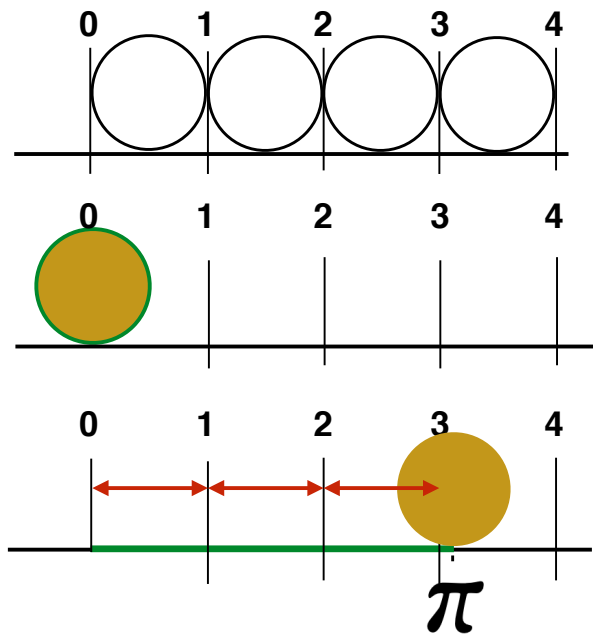
6711136990 8658516398 3150197016 5151168517 1437657618 3515565088 4909898959 9823873455 2833163550 7647918535
8932261854 8963213293 3089857064 20465752590 7091548141 6549859461 6371802709 8199430992 4488957571 2828905923
2332609729 9712084433 5732654893 8239119325 9746366730 5836041428 1388303203 8249037589 8524374417 0291327656
1809377344 4030707469 2112019130 2033038019 7621101100 4492932151 6084244485 963769838 9522868478 3123552658
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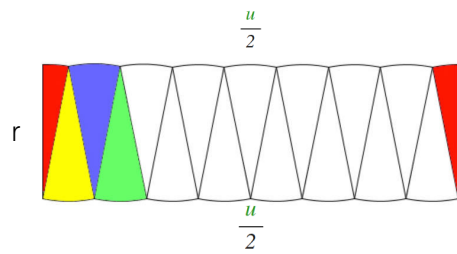
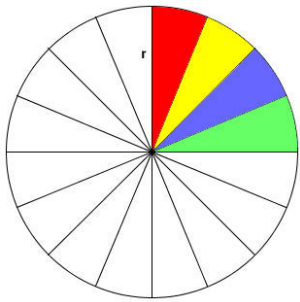
...

...weil keine Definition dich ganz beschreiben kann

π



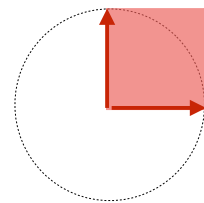




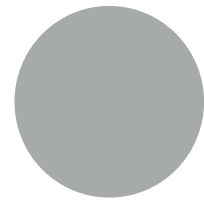
$$A = r * \frac{u}{2}$$

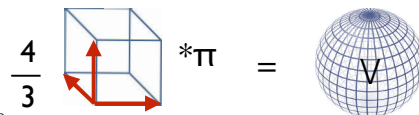
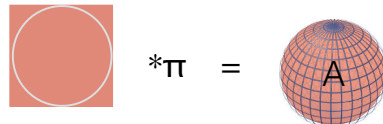
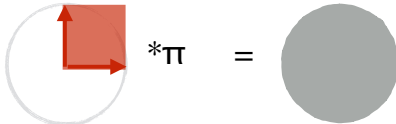
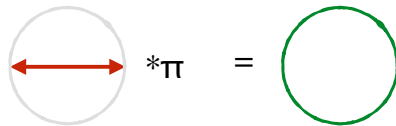
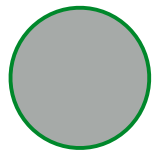
$$A = r * \frac{2 * r * \pi}{2}$$

$$A = r^2 * \pi$$



$*\pi =$





Erinnert dich die 3 an etwas?

Umfang 

$$u = 2 \pi * r$$

Fläche / Area 

$$A = \pi * r^2$$

Oberfläche 

$$A = \pi * d^2$$

$$A = 4 \pi * r^2$$

Rauminhalt/
Volumen 

$$V = \frac{4}{3} \pi * r^3$$